The Decomposed Yoneda–Nyman Proof-Run Theorem

A Formal Conditional Reduction of Track A to Internal Burnol Transport and Blaschke Triviality

Matthew Long
The YonedaAI Collaboration
YonedaAI Research Collective
research@yonedaai.com

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Abstract

We record a formal reduction theorem in the Volume V HoTT–Riemann Hypothesis programme. The new Lean theorem

`yoneda_nyman_decomposed_imported_proof_run_implies_RH`

proves that a decomposed Yoneda–Nyman imported proof run implies both the V6a Track A statement and the V5a classical Riemann Hypothesis proposition. The proof does not assert the Riemann Hypothesis unconditionally. Instead it isolates the exact remaining mathematical burden: internal Yoneda/Mellin transport, transport of the classical Burnol–Beurling–Lax factorization into the internal Yoneda setting, an internal identification of Nyman density with internal Blaschke triviality, regularized operator bridge atoms, and the assertion that the internal Burnol Blaschke factor is trivial. The last assertion is the RH-level zero obstruction. Consequently, a proof of the decomposed proof-run premise would constitute a formal proof of RH in this framework; the theorem itself is a formally verified conditional reduction.

Contents

1 Introduction

The Nyman–Beurling criterion transforms the Riemann Hypothesis into a density problem for fractional-part functions in an L^2 Hilbert space. Burnol's Hardy-space refinement identifies the Mellin image of the Nyman closure with a Blaschke-twisted Hardy submodule, so that the obstruction to density is the inner factor contributed by zeros of ζ in the half-plane $\mathrm{Re}(s) > 1/2$. In classical language, the criterion is an equivalence: density holds if and only if RH holds. It does not prove density.

The Volume V Track A programme tries to internalize this picture in $\mathcal{E}_{\mathrm{cond}}$, connect it to the Connes principal-series object, and feed the resulting density/closure statement into a regularized Hilbert–Pólya operator. The new theorem reported here is the precise formal endpoint of that strategy:

$$\text{decomposed internal proof run} \Longrightarrow \mathrm{RH}_{\mathrm{TrackA}} \wedge \mathrm{RH}_{\mathrm{classical}}.$$

This is a strong result, but it is conditional. Its value is that it removes ambiguity about what remains to be proved. If the decomposed proof-run premise is proved, then the formal RH proposition follows by Lean type checking.

Remark 1.1 (No unconditional RH claim). The theorem does not prove $\mathrm{RH}_{\mathrm{classical}}$ by itself. It proves $\mathrm{RH}_{\mathrm{classical}}$ from a premise containing `InternalBlaschkeTriviality`, which is exactly the internal no-off-critical-zero obstruction. Proving that premise is essentially the RH breakthrough.

2 Classical Analytic Background

2.1 Nyman–Beurling density

Let $\rho(x) = x - \lfloor x \rfloor$, and let χ denote the characteristic function of $(0, 1]$. The classical Nyman–Beurling theorem states, in one standard L^2-normalization, that RH is equivalent to the assertion that χ lies in the closed linear span of fractional-part dilates

$$x \longmapsto \rho\left(\frac{1}{ax}\right), \qquad a \geq 1.$$

Báez-Duarte strengthened this statement by replacing the real dilation parameter with natural dilations $a \in \mathbb{N}$ [5]. In Lean this classical import is represented by the named value

`classical_nymanBeurlingTheorem_import`.

It is deliberately not installed as a global instance.

2.2 Burnol's Blaschke factor

Burnol's refinement [3, 4] places the Nyman problem in a Hardy space on the critical half-plane. The Mellin transform sends fractional-part generators to zeta-multiples, and the closed Nyman subspace corresponds to a Hardy submodule twisted by the Blaschke product attached to zeros of ζ with $\mathrm{Re}(s) > 1/2$. Thus the Blaschke factor is the analytic obstruction: if it is trivial, the Nyman span is dense; if it is non-trivial, density fails.

The literature supports the following kinds of classical theorem imports:

(i) the Mellin identity for fractional-part atoms;

(ii) the Beurling–Lax/Hardy factorization of the closed classical Nyman span;

(iii) the equivalence between classical Nyman density and the absence of zeros in $\mathrm{Re}(s) > 1/2$.

It does not supply the assertion that the Blaschke factor is trivial. That assertion is equivalent in strength to RH.

3 The Lean Interface

3.1 External classical imports

The file

`lean/Vol5/YonedaNymanTrackA/ExternalBurnolImports.lean`

records only the external classical analytic material:

`external_burnol_mellin_nyman_atom_identity` : `MellinNymanAtomIdentity`,
`external_burnol_beurling_lax_factorization` : `ClassicalBurnolFactorization`.

The file intentionally does not axiomatize

`NymanDensity`, `InternalBlaschkeTriviality`, `RH_classical`.

It also no longer imports `NymanDensity` ↔ `InternalBlaschkeTriviality` as an external theorem, because both sides are repo-internal propositions.

3.2 The internal frontier

The decomposed proof-run premise is the structure

`DecomposedYonedaNymanImportedProofRun`

in

`lean/Vol5/YonedaNymanTrackA/TransportDecomposition.lean`.

It contains the following fields:

```
mellin                          : InternalYonedaMellinTransport,
factorization                   : ClassicalBurnolToYonedaTransport,
conservativity                  : BlaschkeConservativityTransport,
realization                     : BlaschkeRealizationTransport,
regularizedCoreBoundary         : RegularizedCoreBoundaryTransport,
regularizedResolvent            : RegularizedResolventTransport,
regularizedSpectrum             : RegularizedSpectrumTransport,
regularizedSelfAdjoint          : RegularizedSelfAdjointTransport,
regularizedClosureReflection    : RegularizedClosureReflectionTransport,
blaschke_trivial                : InternalBlaschkeTriviality.
```

The field `blaschke_trivial` is the decisive RH-level input.

Definition 3.1 (Internal Blaschke triviality)**.** The proposition `InternalBlaschkeTriviality` is the internal assertion that the Burnol Blaschke factor attached to off-critical-line zeta zeros is trivial. Analytically, this says that the inner factor contributed by zeros in $\mathrm{Re}(s) > 1/2$ is absent.

3.3 The regularized operator split

The earlier coarse field

```
InternalBlaschkeTriviality -> RegularizedOperatorBridge
```

was too compressed. The new interface splits it into five auditable atoms:

`RegularizedCoreBoundaryTransport`
closure supplies the domain core, graph-closure, and boundary-condition pieces;

`RegularizedResolventTransport`
closure supplies norm-resolvent/bounded-transform convergence;

`RegularizedSpectrumTransport`
closure supplies the spectral identification;

`RegularizedSelfAdjointTransport`
the assembled regularized premise gives internal self-adjointness of $\mathcal{T}$;

`RegularizedClosureReflectionTransport`
self-adjointness of $\mathcal{T}$ reflects back to Nyman closure.

These assemble into the existing bridge by the theorem

`regularized_operator_bridge_of_decomposed`.

This records the norm-resolvent/bounded-transform route rather than raw operator-norm convergence of wedge projections.

4 The Formal Theorem

Theorem 4.1 (Decomposed Yoneda–Nyman proof-run theorem)**.** *In the Lean development, the theorem*

`yoneda_nyman_decomposed_imported_proof_run`
`_implies_RH`

has type

$$\mathtt{DecomposedYonedaNymanImportedProofRun} \to \left(\mathtt{RH\_TrackA} \wedge \mathtt{RH\_classical}\right).$$

Proof. The proof is formal and compositional.

First, the decomposed regularized operator fields assemble into the existing regularized bridge:

`regularized_operator_bridge_of_decomposed`.

This theorem constructs the bridge premise from closure by combining the core/boundary, resolvent, and spectrum fields, and then supplies the self-adjointness and closure-reflection maps.

Second, the decomposed internal transport fields assemble into the coarser transport package

`InternalYonedaBurnolTransport`

by

$$\texttt{internal\_yoneda\_burnol\_transport\_of\_decomposed}.$$

This package contains the Yoneda/Mellin image comparison, classical-to-internal Burnol factorization transport, internal density/Blaschke identification, Blaschke-to-conservativity transport, Blaschke-to-realization transport, and the regularized operator bridge.

Third, the assembled internal transport package and the Blaschke-triviality witness give

$$\texttt{YonedaNymanImportedProofRun}$$

by

$$\texttt{imported\_proof\_run\_of\_decomposed}.$$

The imported proof run then maps to the older proof-run object `YonedaNymanProofRun`, which packages a `BurnolFactorizationBridge` and the same Blaschke-triviality witness.

Finally, the already verified theorem

$$\texttt{yoneda\_nyman\_imported\_proof\_run\_implies\_RH}$$

applies to this imported proof run and returns

$$\texttt{RH\_TrackA} \wedge \texttt{RH\_classical}.$$

This is exactly the conclusion of Theorem 4.1. □

Corollary 4.2 (Classical RH projection)**.** *The theorem*

$$\textit{RH\_classical\_of\_decomposed\_imported\_proof\_run}$$

extracts *RH_classical* *from a proof of* *DecomposedYonedaNymanImportedProofRun.*

Corollary 4.3 (Track A projection)**.** *The theorem*

$$\textit{RH\_TrackA\_of\_decomposed\_imported\_proof\_run}$$

extracts *RH_TrackA* *from the same proof-run premise.*

5 Why This Would Essentially Prove RH

The new theorem is not merely a slogan. Its consequent is the repo's formal classical RH proposition:

$$\texttt{Vol5.V5aTrackAFormulation.RH\_classical}.$$

Therefore, if one constructs

$$h : \texttt{DecomposedYonedaNymanImportedProofRun},$$

then the expression

$$\texttt{RH\_classical\_of\_decomposed\_imported\_proof\_run h}$$

is a Lean-checked proof of the formal classical RH target.

This is the sense in which proving the decomposed proof-run premise would essentially prove RH in this framework. The caveat is crucial: the premise includes `InternalBlaschkeTriviality`, so constructing it is not routine bookkeeping. It is the mathematical content that classically says there is no Blaschke obstruction from zeros in $\mathrm{Re}(s) > 1/2$.

Proposition 5.1 (The remaining burden is RH-level)**.** *The field*

$$\texttt{blaschke\_trivial : InternalBlaschkeTriviality}$$

in `DecomposedYonedaNymanImportedProofRun` *is equivalent in intent to eliminating the off-critical-line zero obstruction in Burnol's Hardy-space formulation. A proof of this field, together with the internal transport atoms, would close the formal RH route.*

Proof. Burnol's factorization identifies the Nyman obstruction with the inner Blaschke factor attached to zeros of ζ in $\mathrm{Re}(s) > 1/2$. If that factor is trivial, then the Nyman density obstruction disappears. In the Lean development, this passage is represented internally by `BlaschkeRealizationTransport`, `BlaschkeConservativityTransport`, and the regularized operator transports. Once these transports and `InternalBlaschkeTriviality` are supplied, Theorem 4.1 yields the formal RH conclusion. □

6 What Is Not Proved

The theorem does not prove any of the following unconditionally:

`InternalBlaschkeTriviality`, `NymanDensity`, `RH_classical`.

It also does not claim that the external Burnol/Nyman literature proves the internal density/Blaschke equivalence. That equivalence is now correctly placed inside the internal transport frontier.

The remaining irreducible tasks are:

(a) prove `InternalYonedaMellinTransport`;

(b) prove `ClassicalBurnolToYonedaTransport`;

(c) prove the internal density/Blaschke identification in `BlaschkeRealizationTransport`;

(d) prove the conservativity and realization transports;

(e) prove the decomposed regularized operator bridge atoms;

(f) prove `InternalBlaschkeTriviality`.

The final item is the RH-level breakthrough.

7 Blaschke-Defect Frontier and No-Phantom Reduction (Addendum)

This section was added after the original proof-run paper to record a sharper form of the residual obstruction. It does not change the conditional theorem of section 4; it relocates the RH-level breakthrough into a different, more attackable target. RH is still not proved.

7.1 The internal Blaschke defect object

Inside the condensed topos $\mathcal{E}_{\mathrm{cond}}$ we have an internal Hardy object H^2_{int}, and the Burnol classical factorisation gives a Blaschke-twisted submodule

$$B \cdot H^2_{\mathrm{int}} \subseteq H^2_{\mathrm{int}},$$

where B is the internal Burnol Blaschke factor associated to off-critical zeros of ζ. Define the *internal Blaschke defect object*

$$K_B \;:=\; H^2_{\mathrm{int}}/\overline{B \cdot H^2_{\mathrm{int}}}.$$

Classically $K_B = 0$ iff B is trivial iff RH holds. The asymmetry we exploit is between two equivalent but operationally different statements of the same condition:

(1) *Positive existence:* every internal element of H^2_{int} is approximable by rational dilation representables (Nyman density).

(2) *Negative non-existence:* no nonzero element of H^2_{int} is invisible to every rational dilation Yoneda test (no phantom defect).

Statement (1) is the classical literature trap. Statement (2) is the target of the addendum. Although equivalent, (2) admits obstruction-style arguments that (1) does not.

7.2 Detector / invisibility / admissibility decomposition

The Lean module `YonedaNymanTrackA/` now carries an explicit decomposition of the no-phantom claim into three named ingredients. In informal language:

(a) *Detection.* There is a family of finite-rank reproducing kernel objects $\{D_\theta\}_{\theta\in\mathbb{Q}\cap(0,1]}$ inside $\mathcal{E}_{\mathrm{cond}}$, each detecting at most a finite-dimensional slice of H^2_{int}, and jointly probing the rational-dilation representable subuniverse.

(b) *Invisibility.* By construction, K_B is annihilated by every D_θ: the Burnol Blaschke quotient is exactly the part invisible to all rational-dilation tests.

(c) *Admissibility.* The detector family is admissible in $\mathcal{E}_{\mathrm{cond}}$: it is closed under the relevant condensed colimits and interacts with the Hardy submodule structure via Yoneda transport (`InternalYonedaMellinTransport`).

The substantive content is that (a)+(c) suffice to make the rational dilation representables a *generating family* for a sub-cosmos containing H^2_{int}; a phantom defect would then have to live outside the sub-cosmos while still inhabiting H^2_{int}, contradicting admissibility.

7.3 The no-phantom theorem

The Lean target whose proof closes `InternalBlaschkeTriviality` is the following. Write `DetectorSemantics` for the bundle of (a)+(c) above and `BlaschkeDefectObjectIsZero` for the assertion $K_B = 0$ inside $\mathcal{E}_{\mathrm{cond}}$.

Theorem 7.1 (No-phantom Blaschke defect, Lean target)**.** *There is a Lean-checkable implication*

$$\texttt{DetectorSemantics} \implies \texttt{BlaschkeDefectObjectIsZero}.$$

The proof obligation is: cosmos-internal Yoneda detection on the rational-dilation generating family, plus condensed-topos descent on the admissible detector closure, force any phantom element of K_B to vanish.

The Lean module `YonedaNymanTrackA/IrreducibleFrontierSolution.lean` isolates this implication as a normal-form payload: the structure `IrreducibleYonedaNymanFrontierNormalForm` lists the named atoms, and the theorem `irreducible_yoneda_nyman_frontier_of_normal_form` builds `IrreducibleYonedaNymanFrontier` from them. Composition with `yoneda_nyman_irreducible_frontie` yields the classical RH proposition.

7.4 The Burnol-side bridge

The transit from a vanishing internal defect object to internal Blaschke triviality is not a new RH-level claim; it is the internal analogue of a classical Burnol fact.

Theorem 7.2 (Triviality from vanishing defect)**.** *There is a Lean-checkable implication*

$$\mathtt{BlaschkeDefectObjectIsZero} \implies \mathtt{InternalBlaschkeTriviality}.$$

This bridge does not require new RH content. It uses the Mellin commutation and Hardy submodule extensionality lemmas already developed in `MellinCommutation.lean`, `MellinTransportSuccess.lean`, and `HardySubmoduleExtensionality.lean`.

7.5 Relocation of the residual gap

Combining theorems 7.1 and 7.2 with the conditional theorem of section 4 produces the chain

$$\mathtt{DetectorSemantics} \implies \mathtt{BlaschkeDefectObjectIsZero} \implies \mathtt{InternalBlaschkeTriviality} \implies \mathrm{RH}_{\text{classic}}$$

modulo the previously identified internal transport atoms (Mellin transport, classical-Burnol-to-Yoneda transport, conservativity, realisation, and the regularised operator bridge). The residual RH-level burden is therefore no longer the assertion "the internal Burnol Blaschke factor is trivial". It has become the assertion

$$\mathtt{DetectorSemantics} \quad \equiv \quad \text{"a canonical finite-rank RKHS / Yoneda detector family is admissible in } \mathcal{E}_{\text{cond}}\text{"}.$$

Both statements are RH-equivalent. They are not the same proof obligation. The first is a positive density obligation in an L^2 Hilbert space. The second is a generation-and-descent obligation in the condensed topos. The second is the proper home for V1's cosmos-internal Yoneda machinery and Infra-3's analytic-continuation identity theorem; the first is not.

7.6 What the addendum does and does not claim

Remark 7.3 (Honesty contract)*.* This addendum does *not* prove `DetectorSemantics`. It constructs the conditional implication `DetectorSemantics` $\Rightarrow$ $\mathrm{RH}_{\text{classical}}$ and isolates `DetectorSemantics` as a single, named, RH-equivalent obligation that is structurally different from classical Nyman density.

Remark 7.4 (The forbidden shortcut)*.* Constructing `DetectorSemantics` by importing classical Nyman density and projecting it into $\mathcal{E}_{\text{cond}}$ would close the chain trivially but is forbidden: it merely re-imports the literature's RH-equivalent obstruction. The honest version of the obligation requires building the detector family using condensed/categorical resources (V1, Infra-3, Infra-5) without smuggling in an analytic density assumption.

8 Formal Verification

The Lean modules involved are:

`Basic.lean`	Nyman approximants and the L^2 interface,
`Criterion.lean`	classical Beurling–Nyman criterion wrapper,
`Yoneda.lean`	internal dilation category and conservativity inter
`OperatorBridge.lean`	regularized operator bridge,
`BurnolFactorization.lean`	Mellin/Hardy/Blaschke surface,
`ExternalBurnolImports.lean`	external classical Burnol imports,
`TransportDecomposition.lean`	decomposed proof-run theorem,
`MellinCommutation.lean`	Mellin commutation lemmas,
`MellinTransportSuccess.lean`	Mellin transport completeness,
`HardySubmoduleExtensionality.lean`	Hardy submodule extensionality,
`FractionalPartExternalization.lean`	fractional-part externalisation,
`IrreducibleFrontier.lean`	irreducible-frontier structure,
`IrreducibleFrontierSolution.lean`	normal-form solver for the frontier,
`Breakthrough.lean`	audit-friendly breakthrough target,
`ProofSprintAuditD.lean`	sprint audit anchor,
`ProofRun.lean`, `Obstruction.lean`, `TrackABridge.lean`	glue and obstruction summary,
`AnalyticImports.lean`	classical-side analytic imports.

The checked build command is:

`lake build Vol5`.

At the time of writing, this command succeeds for the root `Vol5` target, and the Yoneda–Nyman subtree contains no `sorry`, no `admit`, and no definition of the hard targets as `True`.

9 Conclusion

The decomposed Yoneda–Nyman proof-run theorem is a formally verified conditional RH reduction. Its mathematical meaning is:

$$\boxed{\text{internal Yoneda/Burnol transport} \;+\; \text{internal Blaschke triviality} \implies \text{RH.}}$$

This does not solve RH. It names the exact theorem whose proof would solve RH within the present Track A framework. The contribution is the formal reduction and the decomposition of the remaining burden into auditable internal transport atoms plus the RH-level Blaschke-triviality assertion.

The addendum of section 7 sharpens the residual obstruction further. Internal Blaschke triviality is now derived from a vanishing internal Blaschke defect object, which is in turn derived from the assertion that a canonical finite-rank RKHS / Yoneda detector family is admissible in $\mathcal{E}_{\mathrm{cond}}$. The RH-level burden has therefore been relocated from a positive density obligation in $L^2(0,1)$ to a generation-and-descent obligation in the condensed topos. This is a different obligation from classical Nyman density, structurally suited to V1's cosmos-internal Yoneda machinery and Infra-3's identity theorem. RH remains unproved; the attack surface is sharper.

References

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